Name: _____

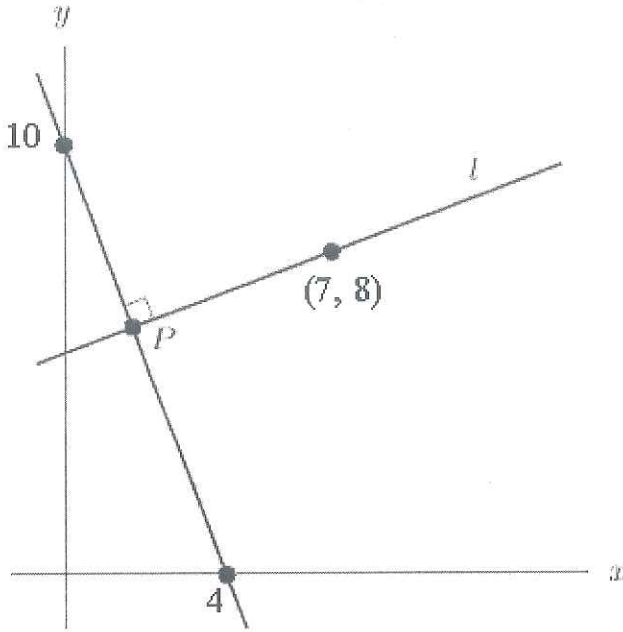**All answers must be justified with work. No work, no credit!**

1. Find the slope of the line l shown n the following figure.

[handwritten, red ink]

SLOPE OF OTHER LINE THROUGH $(4,0)(0,10)$

$M = \dfrac{\Delta y}{\Delta x} = \dfrac{10-0}{0-4} = \dfrac{10}{-4} = -\dfrac{5}{2}$

SLOPE OF l is negative reciprocal

$$\boxed{M = \dfrac{2}{5}}$$

[graph showing y-axis with 10 marked, point (7,8), point P, and x-axis with 4 marked, line l]

2. The following chart gives the number of students in a class that are a specific height in inches

height	55 inches	60 inches	65 inches	70 inches	75 inches
number of students	5	6	6	1	0

[handwritten, red ink] $60 \to 6$ OKAY $65 \to 6$

a) Is the number of students in each category a function of the height? *[handwritten]* YES, EACH X HAS EXACTLY ONE Y-VALUE

b) Is the height in each category a function of the number of students in that category?

[handwritten] NO, AN X-VALUE HAS 2 Y-VALUES $6 \to 60$ $\to 65$ NOT OKAY

3. Are the lines $y = -0.5x + 1$ and $y = 0.5x - 5$ parallel, perpendicular, or neither?
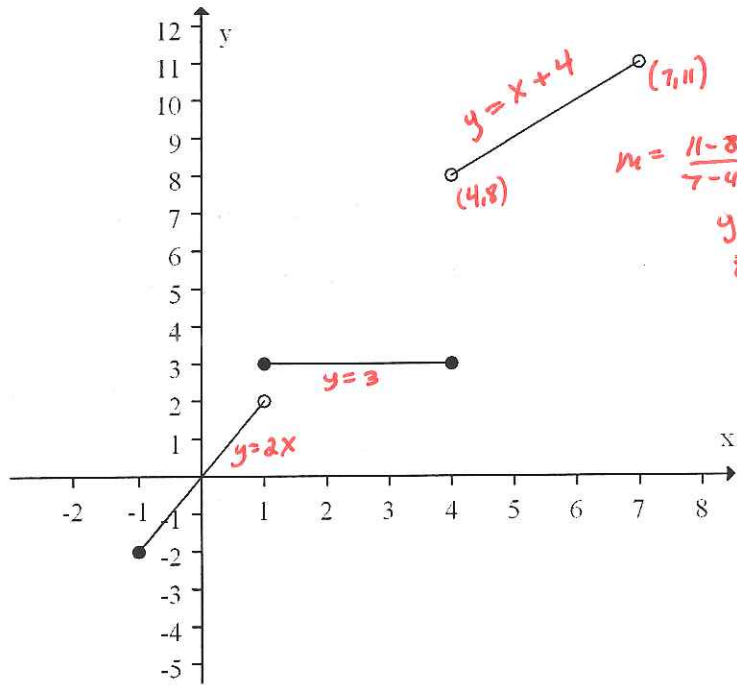
[handwritten] SLOPE $= -\dfrac{1}{2}$ SLOPE $= \dfrac{1}{2}$ NEITHER   NOT THE SAME (PARALLEL) NOT NEGATIVE RECIPROCALS (PERP)

4. Which of the following are *not* in the domain of $r(x) = \dfrac{1}{(x+7)^2} + \sqrt{1-x}$?

[handwritten] MUST BE POSITIVE

A) $x = 7$

B) $x = -7$ *[circled]*

C) $x = 1$ *[circled]*

D) All numbers x such that $x > 1$ *[circled]*

E) All numbers x such that $x < 1$

[handwritten] $(x+7)^2 \neq 0$ $x+7 \neq 0$ $x \neq -7$

$1 - x > 0$ $-x > -1$ $x < 1$ ← ARE IN THE DOMAIN $x > 1$ ← ARE NOT

5. Find a formula for the following graph:

$$y = \begin{cases} 2x & \text{if } -1 \le x < 1 \\ 3 & \text{if } 1 \le x \le 4 \\ x+4 & \text{if } 4 < x < 7 \end{cases}$$

(handwritten annotations on graph)
$y = x+4$ $(7,11)$
$(4,8)$
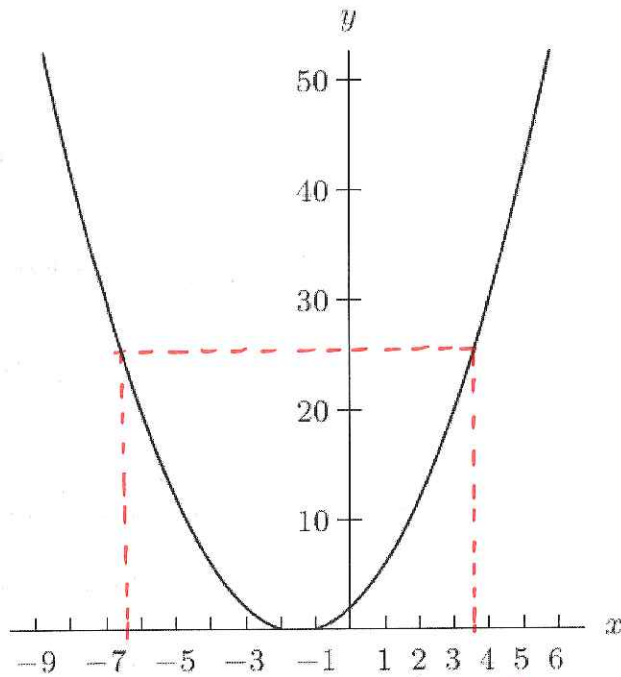$m = \dfrac{11-8}{7-4} = \dfrac{3}{3} = 1$
$y = mx+b$
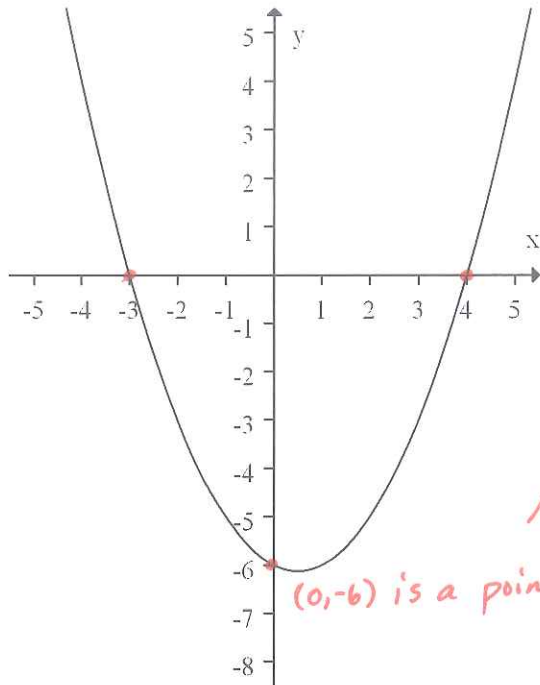$8 = 1(4) + b$
$8 = 4 + b$
$4 = b$
$y = 3$
$y = 2x$

6. Use the graph of $f(x) = x^2 + 3x + 2$ given below to estimate $f^{-1}(25)$. (Mark all correct answers)

find x when y is 25

A) 3.5   B) – 3.5   C) 6.5   D) – 6.5   E) – 1.5   F) 1.5

(A and D are circled)

7. Find a formula for the parabola

ZEROS: $4, -3$

FACTORS: $(x-4)(x+3)$

$$y = a(x-4)(x+3)$$

$$-6 = a(0-4)(0+3)$$

$$-6 = a(-4)(3)$$

$$-6 = -12a$$

$$\frac{1}{2} = a$$

$$\boxed{y = \frac{1}{2}(x-4)(x+3)}$$

$(0,-6)$ is a point

8. Given $f(x) = 4x^2 - 2$ and $g(x) = -3x + 1$, find $f(g(4))$ and $g(f(4))$.

$$g(4) = -3(4) + 1$$
$$= -12 + 1$$
$$g(4) = -11$$
$$f(g(4)) = f(-11) = 4(-11)^2 - 2$$
$$= 4(121) - 2$$
$$= 484 - 2$$
$$\boxed{f(g(4)) = 482}$$

$$f(4) = 4(4)^2 - 2$$
$$= 4(16) - 2$$
$$= 64 - 2$$
$$= 62$$

$$g(f(4)) = g(62) = -3(62) + 1$$
$$= -186 + 1$$
$$\boxed{g(f(4)) = -185}$$

9. What is the equation of the parabola that is concave down, has vertex $(-1, 5)$ and contains the origin.

VERTEX FORM: $y = a(x-h)^2 + k$
$$y = a(x+1)^2 + 5$$
$$0 = a(0+1)^2 + 5 \quad \leftarrow \text{Substitute } (0,0) \text{ for } x \text{ and } y$$
$$0 = a + 5$$
$$-5 = a$$

$$\boxed{y = -5(x+1)^2 + 5}$$

10. Complete the square in order to write the following function in vertex form: $f(x) = x^2 + 2x - 10$.

$$y = x^2 + 2x \qquad -10$$
$$= x^2 + 2x + 1 \leftarrow -1 - 10 \quad \rightarrow \text{BALANCE}$$
$$\boxed{y = (x+1)^2 - 11}$$
$$\left(\frac{b}{2}\right)^2$$

11. The formula for the exponential function P such that $P(3) = 24$ and $P(9) = 6$ is given by $P(t) = $ ___48.00___ (___.79___)^t. Give both answers to 2 decimal places.

$$y = ab^x$$
$$24 = ab^3$$
$$6 = ab^9$$
$$4 = b^{-6}$$
$$4 = \frac{1}{b^6}$$

$$4b^6 = 1$$
$$b^6 = \frac{1}{4}$$
$$b = \left(\frac{1}{4}\right)^{\frac{1}{6}}$$
$$b = .7937$$

$$y = ab^x$$
$$24 = a(.7937)^3$$
$$\frac{24}{.7937^3} = a$$
$$48 = a$$

12. Write a formula that gives the value in an account after t years. Assume that the initial value in the account is $1500 and that the account doubles in value every 10 years.

$$y = ab^t$$
$$y = 1500 \cdot 2^{\frac{t}{10}}$$

13. Kathleen opens a savings account with $1300. The account earns 3.2% annual interest compounded quarterly. How much will be in the account after 13 years?

$$y = 1300\left(1 + \frac{.032}{4}\right)^{13 \cdot 4}$$
$$\boxed{y = \$1967.39}$$

14. Let $P(t) = 500e^{0.05t}$ give the size of a population of animals in year t. After how many years will the population be approximately 1007?

$$1007 = 500e^{.05t}$$
$$\frac{1007}{500} = e^{.05t}$$
$$\ln\left(\frac{1007}{500}\right) = \ln\left(e^{.05t}\right)$$

$$\frac{\ln\left(\frac{1007}{500}\right)}{.05} = \frac{.05t}{.05}$$
$$14.002 = t$$
$$\boxed{\text{Approximately 14 years}}$$

15. Rewriting $e^{3a} = b$ using logs gives

A) $\ln a = b/3$
B) $\ln(b/3) = a$
C) $\ln 3a = b$
D) $\ln b = 3a$ ⟵ (circled)

$$\log_e b = 3a$$
$$\ln b = 3a$$

16. Let $n = \log p$ and $m = \log q$. What is $\log \dfrac{p^3}{q^6}$?

A) $\dfrac{n^3}{m^6}$

B) $\left(\dfrac{n}{m}\right)^{-3}$

C) $(n-m)^{-3}$

D) $3n - 6m$

$\log\left(\frac{p^3}{q^6}\right) = \log(p^3) - \log(q^6)$

$= 3\log p - 6\log q$

$= 3n - 6m$

17. The doubling time for a bank account that is growing by 5.1% per year (compounded continually) is **13.6** years. Round 1 decimal place.

$A = A_0 e^{.051t}$

$2A_0 = A_0 e^{.051t}$

$2 = e^{.051t}$

$\ln 2 = \ln(e^{.051t})$

$\dfrac{\ln 2}{.051} = \dfrac{.051t}{.051}$

$t = 13.591$

$t = 13.6 \text{ years}$

18. What is the effect of the translation $f(x - 6a) + b$ on the graph of the function $f(x)$? Assume a and b are positive constants.

A) Shift right $6a$, then up b.
B) Shift right $6a$, then down b.
C) Shift left $6a$, then up b.
D) Shift left $6a$, then down b.

right $6a$ up b

19. The graph of a function f has been shifted down 4 units, shifted 5 units to the right, and then stretched vertically by a factor of 8. The new graph is produced by a function g. Find a formula for g in terms of f.

$g(x) = f(x) - 4$

$= f(x-5) - 4$

$= 8(f(x-5) - 4)$

$g(x) = 8f(x-5) - 32$

20. Find the coordinates of the point on the unit circle with angle α if $\cos \alpha = 0.630$. Round each coordinate to 3 decimal places.

ANY COORDINATE ON A CIRCLE $(r\cos\theta, r\sin\theta) = \left(.630, \sin\left(\cos^{-1}(.630)\right)\right)$
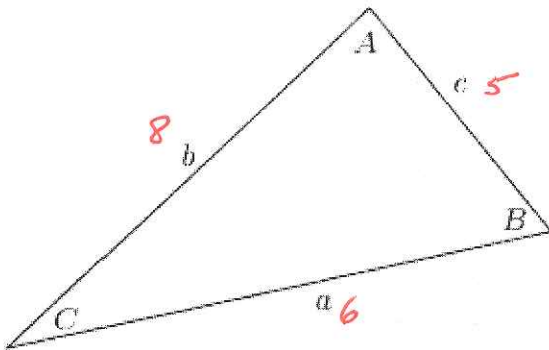
$\alpha = \cos^{-1}(.630)$

$(.630, .777)$

21. Find the coordinates of the point at angle $-23°$ on a circle of radius 7.1. Round each coordinate to 3 decimal places, if necessary.

$(r\cos\theta, r\sin\theta)$

$(7.1\cos(-23°), 7.1\sin(-23°))$

$(6.536, -2.774)$

22. Find the measures of the angles of the triangle if $a = 6$, $b = 8$, and $c = 5$. Round to two decimal places.

LARGEST ANGLE FIRST

$b^2 = a^2 + c^2 - 2ac\cos B$

$8^2 = 6^2 + 5^2 - 2(6)(5)\cos B$

$64 = 61 - 60\cos B$

$3 = -60\cos B$

$-\frac{3}{60} = \cos B$

$B = \cos^{-1}\left(-\frac{3}{60}\right)$

$\boxed{B = 92.87°}$

$\frac{\sin 92.86598}{8} = \frac{\sin A}{6}$

$\frac{8\sin A}{8} = \frac{6(\sin 92.86598)}{8}$

$\sin A = \frac{6\sin 92.86598}{8}$

$A = \sin^{-1}\left(\frac{6\sin 92.86598}{8}\right)$

$\boxed{A = 48.51°}$

$C = 180 - 92.87 - 48.51$

$\boxed{C = 38.62°}$

23. What is the amplitude, period, and midline of the periodic function $y = 2\cos(2x) - 8$?

$P = \frac{2\pi}{B}$

$= \frac{2\pi}{2}$

$P = \pi$

AMPLITUDE = 2
PERIOD = π
MIDLINE: $y = -8$

$y = 2\cos(2x) - 8$

AmP   B   Midline

24. The angle $135°$ is equivalent to $\underline{\frac{3}{4}}$ π radians.

$\frac{135}{180} = \frac{\theta}{\pi}$

$\frac{180\theta}{180} = \frac{135\pi}{180}$

$\theta = \frac{27\pi}{36}$

$\theta = \frac{3\pi}{4}$

25. What is the length of an arc cut off by an angle of $210°$ in a circle of radius 2.9 meters? Give your answer correct to 3 decimal places.
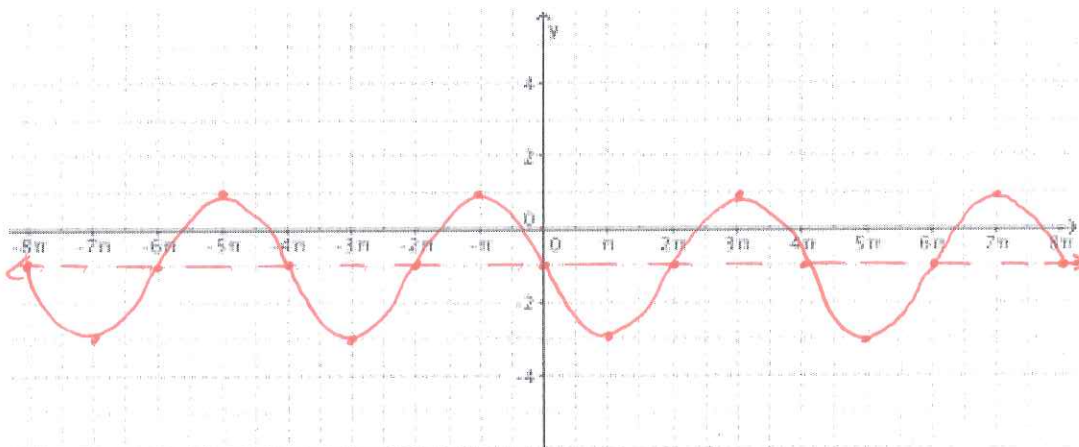
$\frac{210°}{360°} = \frac{S}{2\pi(2.9)}$

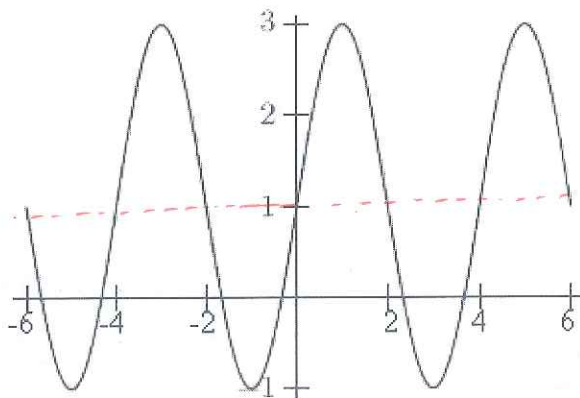$\frac{360S}{360} = \frac{210(2\pi)(2.9)}{360}$

$S = 10.629 \text{ meters}$

26. For the following function, identify the amplitude, period, horizontal shift, and vertical shift and then graph it from $-8\pi \le t \le 8\pi$:

AMP: 2   Period $= \dfrac{2\pi}{B}$

$\quad = \dfrac{2\pi}{\frac{1}{2}}$

$\quad = 4\pi$

$$f(t) = 2\sin\left(\frac{1}{2}t - \pi\right) - 1$$

HOR SHIFT $= \dfrac{-C}{a} = \dfrac{\pi}{\frac{1}{2}} = 2\pi$

VERT SHIFT: -1

27. The formula for the following trigonometric function is $f(t) = \underline{\ 2\ }\sin(\underline{\ \frac{1}{2}\ }\pi t) + \underline{\ 1\ }$.

Amp $= 2$

Period $= 4$

Midline: $y = 1$

$P = \dfrac{2\pi}{B}$

$\dfrac{4}{1} = \dfrac{2\pi}{B}$

$4B = 2\pi$

$\quad B = \dfrac{2\pi}{4} = \dfrac{\pi}{2}$

28. Find a formula for a deer population which oscillates over a 6 year period between a low of 1000 in year $t = 0$ and a high of 3500 in year $t = 3$.

Reflected
Cosine – starts
down

midline: $y = \dfrac{1000 + 3500}{2}$

$\quad y = 2250$

Amp $= \dfrac{3500 - 1000}{2}$

Amp $= 1250$

$P = \dfrac{2\pi}{B}$

$\dfrac{6}{1} = \dfrac{2\pi}{B}$

$6B = 2\pi$

$\quad B = \dfrac{2\pi}{6} = \dfrac{\pi}{3}$

$\boxed{y = -1250\cos\left(\frac{\pi}{3}t\right) + 2250}$

$\uparrow$
Reflection

29. A ferris wheel sitting on the ground is 24 meters in diameter and makes one revolution every 7 minutes. If you start in the 9 o'clock position $t = 0$ and the wheel is rotating clockwise, write a formula for your height above the ground at time t.

Midline: $y = 12$ Period $= 7$ min   Amplitude: $\frac{24}{2} = 12$

$P = \frac{2\pi}{B}$ $B = \frac{2\pi}{7}$

$\frac{7}{1} = \frac{2\pi}{B}$

$7B = 2\pi$

$\boxed{y = 12\sin\left(\frac{2\pi}{7}t\right) + 12}$

30. For positive numbers x, what is the inverse of $h(x) = e^{\sqrt{x}-7}$?

$y = e^{\sqrt{x}-7}$

$x = e^{\sqrt{y}-7}$

$\ln x = \ln\left(e^{\sqrt{y}-7}\right)$

$\ln x = \sqrt{y} - 7$

$\boxed{\ln x + 7 = \sqrt{y}}$

$\boxed{(\ln x + 7)^2 = y}$

$h^{-1}(x) = (\ln x + 7)^2$

31. Let $g(x) = \frac{3}{x} + 6$. Use composition of functions to check/prove that $g^{-1}(x) = \frac{3}{x-6}$.

$g(g^{-1}(x)) = \frac{3}{\frac{3}{x-6}} + 6$

$= 3 \cdot \frac{x-6}{3} + 6$

$= x - 6 + 6$

$= x \checkmark$

$g^{-1}(g(x)) = \frac{3}{\frac{3}{x} + 6 - 6}$

$= \frac{3}{\frac{3}{x}}$

$= 3 \cdot \frac{x}{3}$

$= x \checkmark$

32. Given $f^{-1}(x) = 300(1.03)^x$, solve $f^{-1}(x) = 350$. Round to 3 decimal places.

$350 = 300(1.03)^x$

$\frac{350}{300} = 1.03^x$

$\log\left(\frac{7}{6}\right) = \log(1.03^x)$

$\frac{\log\left(\frac{7}{6}\right)}{\log 1.03} = \frac{x \log 1.03}{\log 1.03}$

$\boxed{5.215 = x}$

33. The power function through the point (2, 5) and (8, 12) is $y = kx^p$, where $k = \underline{3.227}$ and $p = \underline{.631}$. Round the second answer to 3 decimal places.

$y = Kx^p$

$12 = K(8)^p$

$5 = K(2)^p$

$\frac{12}{5} = 4^p$

$\log\left(\frac{12}{5}\right) = \log 4^p$

$\frac{\log\left(\frac{12}{5}\right)}{\log 4} = \frac{p \log 4}{\log 4}$

$.6315172 = p$

$.631 = p$

$y = kx^p$

$5 = K(2)^{.6315172}$

$\frac{5}{2^{.6315172}} = \frac{K(2)^{.6315172}}{2^{.6315172}}$

$3.22749 = k$

$3.227 = k$

Page 8

34. Let $f(x)=(x-2)^2(x+4)$. Which of the following figures shows the graph of $f(-0.5x)$?

HORIZONTAL STRETCH SF 2

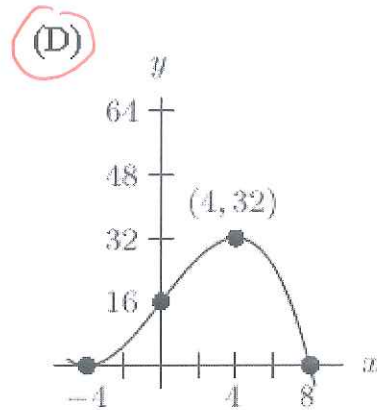HORIZONTAL REFLECTION

(A)

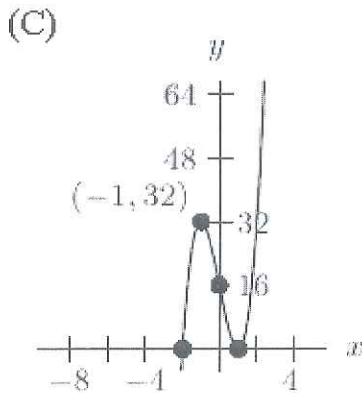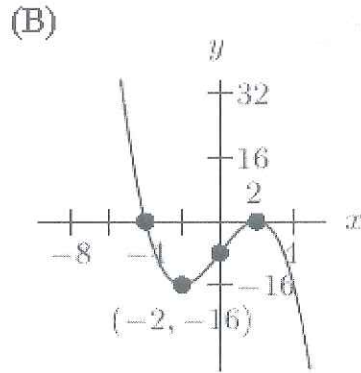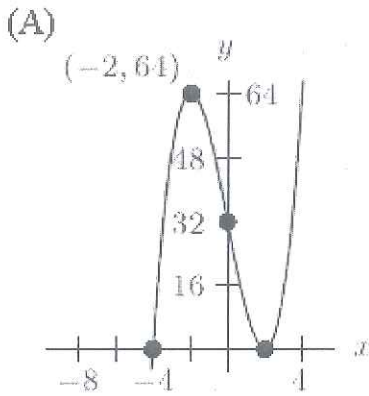
$(-2,64)$

(B)

$(-2,-16)$

Reflect!

$x=2 \rightarrow x=-2$

$x=-4 \rightarrow x=4$

STRETCH SF 2

$x=-2 \rightarrow x=-4$

$x=4 \rightarrow x=8$

(C)

$(-1,32)$

(D)

$(4,32)$

35. Which of the following are polynomials:

A) $y=15x^4$

B) $y=(x^2+2)(x-15)e^x$ EXPONENTIAL

C) $y=1-15t+\sqrt{2t^5}$ FRACTIONAL EXPONENT

D) $y=1-15t+\sqrt{2}t^5$

36. Compute the following limits:

a) $\lim_{x\to\infty}\left(-5x^4+7x^3-116x^2\right)$ $\lim_{x\to\infty} -5x^4 = -\infty$

b) $\lim_{x\to-\infty}\left(-5x^4+7x^3-116x^2\right)$ $\lim_{x\to-\infty} -5x^4 = -\infty$

37. Find all intercepts, zeros, asymptotes, and holes of: $f(x) = \dfrac{x^2 - 25}{x^2 + 6x}$?

$f(x) = \dfrac{(x+5)(x-5)}{x(x+6)}$

ZEROS $x+5=0$ $x-5=0$

$\boxed{x=-5 \qquad x=5}$

VA: $\boxed{x=0}$ $x+6=0$

$\boxed{x=-6}$

$f(0) = \dfrac{(0+5)(0-5)}{0(0+6)}$

undefined, no y-intercept

HA: $\dfrac{x^2}{x^2} = \boxed{y = 1}$

NO HOLES

38. As $x \to +\infty$, $f(x) = -4x^3 + 6x^2 + 17 \to \underline{\ -\infty\ }$. Enter "infinity" or "- infinity" for ∞ or $-\infty$.

as $x \to \infty$ $-4x^3 \to -\infty$